Monte Carlo renormalisation group for the true self-avoiding walk

1986 J. Phys. A: Math. Gen. 19 961

(http://iopscience.iop.org/0305-4470/19/6/024)

J. Phys. A: Math. Gen. 19 (1986) 961-971. Printed in Great Britain

Monte Carlo renormalisation group for the true self-avoiding walk

Hans Christian Öttinger

Fakultät für Physik der Universität Freiburg, Hermann-Herder-Str. 3, D-7800 Freiburg, West Germany

Received 22 April 1985

Abstract. Following the general ideas by Swendsen a Monte Carlo renormalisation group (MCRG) estimation of the end-to-end distance exponent ν for the one-dimensional true self-avoiding walk (TSAW) has been carried out.

Because the TSAW is known to reach its asymptotic behaviour only very slowly an approximate expression for the fixed point has been determined by Monte Carlo techniques. This approximate fixed point has then been used as a starting point for the MCRG. The final result for the exponent ν is 0.665 ± 0.004.

1. Introduction

During the last two years a number of generalisations of random walk models have appeared in the literature (Duxbury *et al* 1984, Duxbury and de Queiroz 1985 and references therein). The great interest in these generalised random walk models stems from their unusual critical properties rather than from their relevance to certain physical problems. In the present paper one of these new models—the so-called 'true self-avoiding walk' (TSAW) in one dimension—will be studied by the Monte Carlo renormalisation group (MCRG) method. An approximate expression for the fixed point model which governs the universal behaviour of the TSAW at large distances will be used to estimate the end-to-end distance exponent ν.

The TSAW (Amit *et al* 1983)—which describes the path of a random walker who tries to avoid regions in space previously visited—can be defined in the one-dimensional case as follows: starting at the origin, the random walker has to move at any step to one of the nearest neighbours $i \pm 1$ of the current position i. The probability $p_{i\pm 1}$ for moving to the site $i \pm 1$ depends on the number of previous visits $n_{i\pm 1}$ of these sites by

$$p_{i\pm 1} = \exp(-gn_{i\pm 1})[\exp(-gn_{i-1}) + \exp(-gn_{i+1})]^{-1} \tag{1}$$

where the parameter g defines the strength with which the walk avoids itself. Although the TSAW is commonly studied as a mathematical model in its own right it should be mentioned that special physical realisations of this model have been described by Bulgadaev and Obukhov (1983) and Family and Daoud (1984).

From a self-consistent approach to the TSAW Pietronero (1983) obtained the universal (independent of g for $0 < g < \infty$) end-to-end distance exponent $\nu = 2/3$ (see also Obukhov 1984, Family and Daoud 1984, Öttinger 1985a). This result for the one-dimensional TSAW was confirmed by Monte Carlo simulations (Rammal *et al* 1984, Bernasconi and Pietronero 1984) and by exact enumeration methods (Stella *et al* 1984, Byrnes and Guttmann 1984). The universality of ν has been supported by renormalisation group studies too (De Queiroz *et al* 1984, Obukhov and Peliti 1983, Peliti 1984a, b).

A renormalisation group for random walk models which can be treated by Monte Carlo techniques can be obtained from two different starting points. The first approach, which leads to the real-space renormalisation group, is based on rescaling the size of the lattice on which the random walk takes place. Following this approach one has to determine the change in the number of steps of the random walk under rescaling of the size of the lattice (more precisely, one determines the change in the variable conjugated to the number of steps, namely the fugacity). The details and the results of this approach are reviewed in the article by Stanley *et al* (1982).

In the second approach, which is called renormalisation along the chemical sequence, a fixed number of steps of the original random walk are grouped together to form a single step of the renormalised random walk. Following this approach, which starts with a rescaling of the number of steps or the fugacity, one has to determine the effective length of the renormalised step; this is the change in the scale of length. This renormalisation along the chemical sequence has been suggested by de Gennes (1979).

Ma (1976) suggested a combination of the renormalisation group with Monte Carlo techniques (see also Swendsen 1979, 1982). For both approaches outlined above the combination of the renormalisation group and Monte Carlo methods has been used in order to estimate the end-to-end distance exponent of random walk models (see e.g. Redner and Reynolds (1981) and Kremer *et al* (1981) for applications to the self-avoiding walk). In the present paper the real-space MCRG will be used in order to estimate the end-to-end distance exponent for the TSAW in one dimension.

In § 2 an exactly solvable random walk model will be used in order to explain a canonical renormalisation rule for one-dimensional kinetic walk models and also to discuss several effects observed for the TSAW. In § 3 an approximate expression for the fixed point, to which the TSAW tends under iterated rescaling of length, is estimated by a Monte Carlo simulation. The results of a subsequent simulation of this fixed point model will be used in § 4 in order to estimate the exponent ν. A brief summary concludes the paper.

2. Renormalisation group for the turning point model

Duxbury *et al* (1984) have introduced the 'turning point model' (TPM) in order to study the properties of interacting random walk models. In the present section the real-space renormalisation group for this simple model will be described for two purposes: firstly, the general rules for the construction of renormalised steps for kinetic walk models and for the calculation of the end-to-end distance exponent ν can be explained most clearly for an exactly solvable model. Secondly, the renormalisation group for the TPM will also help us to understand several properties observed for the more interesting TSAW model.

The TPM is a model with short range memory effects only. Consider a one-dimensional random walk in which each step has a different probability according to whether it is in the same direction as (with probability p) or opposite to (with probability $1-p$) the immediately preceding one. Clearly, the case $p=\frac{1}{2}$ corresponds to the ordinary random walk. For any value of p the TPM can be solved by elementary probability theory. For example, one obtains for the average squared end-to-end distance after

N steps the equation

$$\langle R_N^2 \rangle = [p/(1-p)]N - (2p-1)[1-(2p-1)^N]/[2(1-p)^2] \qquad (0 \leq p < 1) \tag{2}$$

from which one reads off the exponent $\nu = \frac{1}{2}$ (equation (2) differs slightly from the formula given by Duxbury *et al* (1984) because these authors have introduced certain boundary conditions). Thus, the TPM displays ordinary random walk behaviour for any value of p. However, due to the factor $p/(1-p)$ in equation (2), the TPM behaves for large N like an ordinary random walk on a lattice with spacing $[p/(1-p)]^{1/2}$ instead of 1. This effect can be interpreted as a decrease (increase) of the diffusion constant for $p < \frac{1}{2}$ ($p > \frac{1}{2}$) due to favouring (suppressing) turning points, thus modifying the 'stiffness' of the walk.

For renormalisation group studies one has to introduce a fugacity in order to control the number of steps instead of treating the model at a fixed value of the number of steps. For kinetic growth models the most reasonable way is to introduce the fugacity K as the probability of taking one step at all (Nakanishi and Family 1984). Then, the probability for a random walk of N steps is given by $K^N(1-K)$ and one obtains the relation

$$\langle N \rangle = K/(1-K) \tag{3}$$

between the average number of steps and the fugacity K. For $K \to K_c = 1$ one of course obtains random walks of infinite number of steps.

For one-dimensional kinetic walk models there exists a very natural procedure of length rescaling. It will now be described how a number of steps of the original model can be grouped together to form a renormalised step upon length rescaling by a factor of s. To this end one passes from the original lattice $L = \{i / i \in \mathbb{Z}\}$ to the coarse grained lattice $L' = \{si / i \in \mathbb{Z}\}$. From the kinetic walk on the original lattice L (starting at the origin) one obtains a renormalised kinetic walk on the lattice L' (also starting at the origin) in the following way: a new renormalised step is completed whenever the original kinetic walk reaches a point belonging to the lattice L'. According to this canonical renormalisation rule a length rescaling by a factor of s_1 followed by a rescaling by a factor of s_2 is exactly equivalent to a single length rescaling by a factor of $s_1 s_2$.

In consequence of the short range memory of the TPM the probability of a renormalised step to the left or to the right depends only on the direction of the immediately preceding renormalised step for this model. Thus, the evolution of the renormalised kinetic walk is determined by rules of precisely the same form as for the original kinetic walk, and the renormalised model can be described exactly by suitable parameters K' and p'. For $s = 2$ these renormalised parameters can be calculated easily by elementary probability theory:

$$K' = pK^2/[1-(1-p)K^2] \tag{4a}$$

$$p' = p/[1-(1-p)(1-2p)K^2]. \tag{4b}$$

The end-to-end distance exponent ν can be obtained from the renormalisation group transformation (4) in the usual way. To begin with, one has to find the critical points K_c and p_c (i.e. the fixed points) of the renormalisation group transformation.

Because $K_c = 1$ for the interesting fixed points one has

$$(1-K')/(1-K) \xrightarrow[K\to K_c]{} (\partial K'/\partial K)|_{K=K_c}$$

and therefore the standard theory (Kogut and Wilson 1974) gives

$$\nu(K, p_c) = \ln 2\left[\ln\left(\frac{1-K'(K, p_c)}{1-K}\right)\right]^{-1} \xrightarrow[K\to K_c]{} \nu(p_c). \quad (5)$$

This formula is very plausible because the argument of the logarithm in the numerator is the ratio of the length scales R/R' while the argument of the logarithm in the denominator is the ratio of the average number of steps $\langle N\rangle/\langle N'\rangle$ (notice that according to equation (3) one has $\langle N\rangle \approx 1/(1-K)$ near $K_c = 1$). Thus, equation (5) implies $R \sim \langle N\rangle^{\nu}$.

For $K_c = 1$ one obtains from equations (4) and (5) the following fixed points and critical exponents governing the scaling behaviour of the TPM:

(i) $p_c = 1$, $\nu = 1$ (self-avoiding walk fixed point)

(ii) $p_c = \frac{1}{2}$, $\nu = \frac{1}{2}$ (pure random walk fixed point).

Figure 1 displays the complete flow diagram for the TPM. The scaling behaviour ($K = 1$) for all values of $p < 1$ is governed by the pure random walk fixed point.

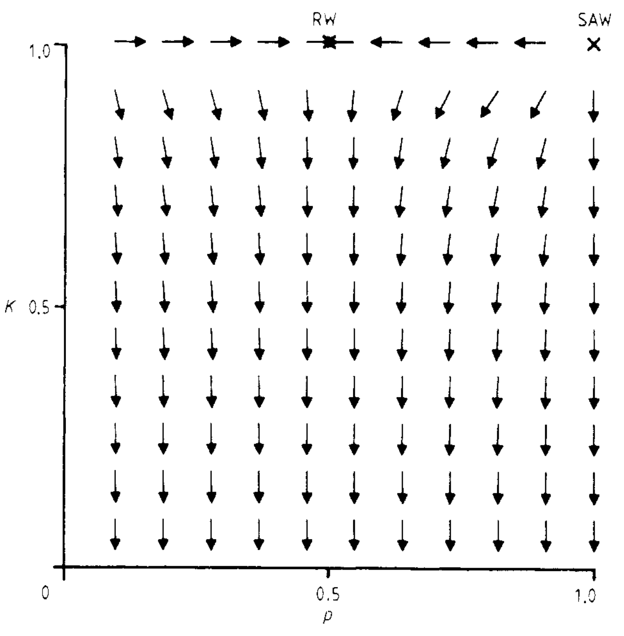

Figure 1. Flow diagram for the one-dimensional TPM. SAW and RW denote the self-avoiding walk and the pure random walk fixed points, respectively.

In the subsequent MCRG study of the TSAW it is impossible to simulate the fixed point model at the critical value $K_c = 1$ (i.e. walks with an infinite number of steps), and one is forced to adopt a proper extrapolation procedure. In figure 2 the exact results for ν (equation (5)) have been plotted against $1/\langle N\rangle = (1-K)/K$ for the TPM at the critical value $p = \frac{1}{2}$. Obviously, one can carry out a linear extrapolation for $1/\langle N\rangle \to 0$. Because the slope in figure 2 only varies very slowly with $1/\langle N\rangle$, the value

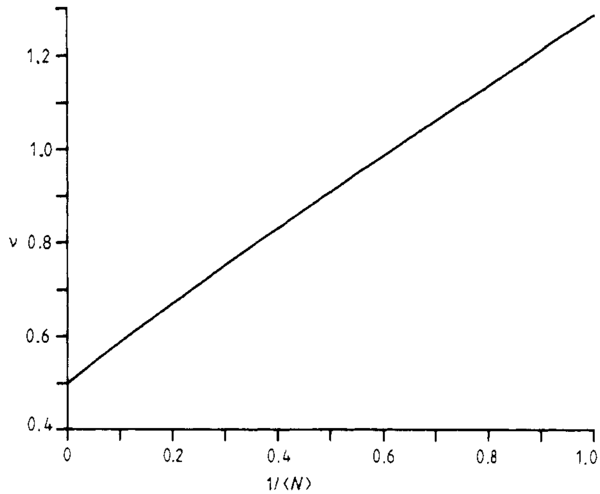

Figure 2. End-to-end distance exponent ν against reciprocal number of steps $1/\langle N\rangle$ for the TPM at the critical value $p=\frac{1}{2}$.

of ν obtained from a linear extrapolation in the entire region $1/\langle N\rangle \leq 1$ is surprisingly close to the exact value $\nu=\frac{1}{2}$. The corresponding figure for the TSAW will display a very similar behaviour and we shall be able to extract the asymptotic value of ν from our results in the range from $\langle N\rangle \approx 3$ to $\langle N\rangle \approx 20$ (notice that in a direct Monte Carlo simulation of the TSAW the asymptotic value of ν can only be estimated from the data for very large values of N, typically of the order of 10^5).

3. Renormalisation of the TSAW

From former Monte Carlo simulations (Rammal *et al* 1984, Bernasconi and Pietronero 1984) it is known that the TSAW reaches its asymptotic behaviour only for very large values of N, that is, the TSAW model is far away from the fixed point which governs its scaling behaviour. For this reason it is useful to look for another model that has the same scaling properties but is nearer to the corresponding fixed point. Using such a model the effort for a MCRG estimation of the exponent ν will be strongly reduced because the fixed point model is already reached after a few successive rescalings. Furthermore, the form of the fixed point model (which depends on the renormalisation rule) is an interesting quantity in its own right because the rescaling is carried out according to the *canonical* renormalisation rule described in the preceding section. For these two reasons an approximate expression for the fixed point model will be determined in the present section.

Because the fixed point is in general expected to be described by an infinite number of parameters one is forced to introduce a reasonable truncation of the parameter space. To this end we consider the following class of one-dimensional kinetic walk models: let i_2 be the current position of the random walker, i_1 the position which the walker visited immediately before, and i_3 the opposite neighbour of the current position i_2. The probabilities for moving in the next step to the sites i_1 or i_3 are defined by

$$p_1 = 1/(1+e^{\Delta E}) \qquad p_3 = 1/(1+e^{-\Delta E}) \tag{6}$$

respectively. In these formulae the quantity ΔE is allowed to depend on the occupation numbers n_{i_1}, n_{i_2} and n_{i_3}. Obviously, the TSAW is obtained for $\Delta E = g(n_{i_1} - n_{i_3})$ (note that in this case i_1 and i_3 are involved in a symmetric form in equation (6)) and the TPM is obtained for $\Delta E = \ln[p/(1-p)]$ (this is $p_1 = 1-p$, $p_3 = p$).

Simulations of the TSAW for various values of the self-avoidance parameter and of the rescaling factor show that the quantity ΔE in equation (6) for the rescaled TSAW can be approximated very well by the expression

$$\Delta E = g(n_{i_2})(n_{i_1} - n_{i_3}) + b(n_{i_2}). \tag{7}$$

The first term corresponds to a generalised TSAW for which the self-avoidance parameter g is allowed to depend on the number of previous visits n_{i_2} (Öttinger 1985b). The quantity $b(n_{i_2}) > 0$ implies that the renormalised TSAW has—just like the TPM—a tendency to suppress turning points, and therefore the renormalised TSAW displays an increased stiffness. This fact is the analogue of the renormalised diffusion constant found by perturbative renormalisation of the TSAW near two dimensions (Amit *et al* 1983, Obukhov and Peliti 1983).

In order to obtain a useful approximation to the fixed point the TSAW has to be renormalised several times one after another. Thus, starting with a renormalised model of the form (6) and (7) the kinetic walk obtained upon a further length rescaling should be of the form (6) and (7), also. Figure 3 displays a check of this requirement, where the model of table 1 has been chosen to be the input model (this is our final approximation to the fixed point and therefore the most interesting check). By a Monte Carlo simulation the parameters $\Delta E(n_{i_1}, n_{i_2}, n_{i_3})$ have been estimated upon a rescaling by a factor of $s = 8$ for which reasonably large values of the occupation numbers could be obtained. Figure 3 shows ΔE against $n_{i_1} - n_{i_3}$ for $n_{i_1} = n_{i_2} = 6$. The results are very close to a straight line. Keeping n_{i_2} fixed the corresponding results for $n_{i_1} = 3$ ($n_{i_1} = 9$) have been included in this figure, where the data points have for reasons of clearness been shifted to the right (left) along the straight line. Figure 3 confirms that for fixed n_{i_2}

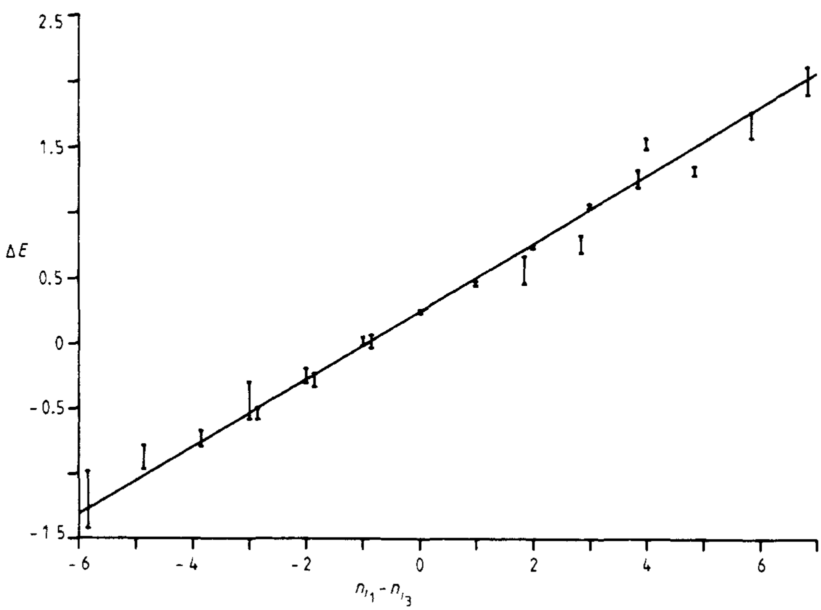

Figure 3. The quantity ΔE introduced in equation (5) against $n_{i_1} - n_{i_3}$ for fixed n_{i_2} and several values of n_{i_1} (see text).

Table 1. The parameters $g(n)$ and $b(n)$ (defining a kinetic walk model according to equations (5) and (6)) for an approximation to the fixed point which governs the scaling behaviour of the TSAW.

n	$g(n)$	$b(n)$
1	0.440	0.575
2	0.482	0.407
3	0.406	0.255
4	0.337	0.247
5	0.290	0.242
$n>5$	$0.056+1.219/n$	0.240

the quantity ΔE only depends through $n_{i_1}-n_{i_3}$ on the variables n_{i_1} and n_{i_3}, and is moreover a linear function of $n_{i_1}-n_{i_3}$.

As a starting point for the estimation of an approximation to the fixed point governing the scaling behaviour of the TSAW a Monte Carlo simulation of the TSAW for $g=0.5$ has been carried out. The quantities $\Delta E(n_{i_2}, n_{i_1}-n_{i_3})$ have been estimated from the transition probabilities (equation (6)) of the rescaled model (for the rescaling factor $s=8$) and the renormalised parameters $g(n)$ and $b(n)$ have been calculated by linear regression according to equation (7). Of course, only walks of finite length can be generated in a computer simulation, and therefore the simulation has been carried out for a fugacity slightly smaller than the critical one, namely $1-K=10^{-4}$. Then, the walks cease after a finite number of steps and the occupation numbers obtained in the simulation for the rescaled walks are bounded (the largest renormalised occupation number obviously decreases with increasing rescaling factor). For this reason the renormalised parameters $g(n)$ and $b(n)$ could be estimated with reasonable accuracy only for $n\leq 20$ for the chosen rescaling factor $s=8$.

The same procedure has been repeated for the kinetic walk model (6) and (7) with the renormalised parameters $g(n)$, $b(n)$. In doing so the values $g(n)$, $b(n)$ for $n\leq 5$ resulting from the first simulation have been used directly while for larger values of n first order expansions in $1/n$

$$g(n)=p_g+(q_g/n) \qquad b(n)=p_b+(q_b/n) \qquad \text{for } n>5 \tag{8}$$

have been assumed to yield good approximations. The results for $n=6$–20 were in good agreement with the expansion (8) and the coefficients p_g, q_g, p_b and q_b could be estimated by linear regression. This procedure was repeated several times until finally the values of the renormalised parameters remained unchanged within the statistical error bars upon further rescaling. This final approximation to the fixed point was reached after six successive rescalings corresponding to an overall rescaling factor of $8^6=262\,144$ (for each rescaling some 90 min of CPU time on a Sperry 1100/82 computer were used). The results for the critical parameters are summarised in table 1.

Figure 4 shows $g(n)$ and $b(n)$ as functions of $1/n$. The continuous lines represent the parameters of table 1, and the data points on these lines are the results for $g(n)$ and $b(n)$ obtained by a further rescaling by a factor of 8 (note that the error bars illustrate the statistical uncertainty of the approximate fixed point of table 1). If the renormalisation of kinetic walk models of the form (6) and (7) led to models of exactly the same form the result for the (accordingly exact) fixed point would be independent of the rescaling factor. In order to give a feeling of the deviation of our approximation

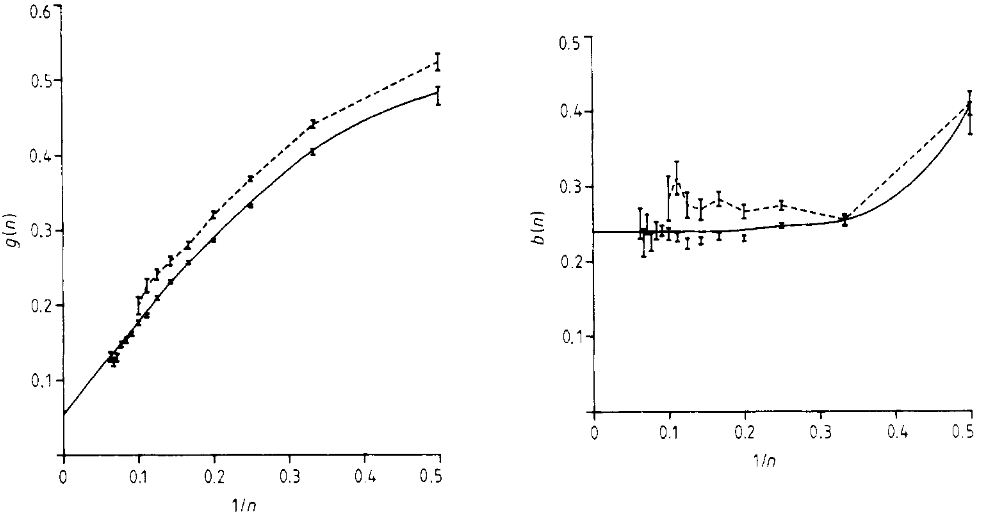

Figure 4. Renormalisation of the parameters $g(n)$ and $b(n)$ for the approximate fixed point defined in table 1 (see text).

(table 1) from the exact fixed point due to the truncation of the number of parameters, the renormalised parameters upon rescaling by a factor of $s = 16$ have been included in figure 1 (the data points connected by broken lines). Small deviations from the $s = 8$ result obviously exist. However, it should be pointed out that these deviations are in principle of no importance for the MCRG study described in the next section because any model in the universality class of the TSAW can be used as a starting point for the MCRG. A model near the fixed point merely reduces the computational effort.

Concerning the properties of the fixed point of the canonical renormalisation rule one can state at least

(i) the self-avoidance parameter g is rather small (except for very small values of the occupation number n)

(ii) the parameter b is greater than zero and therefore leads to a suppression of turning points (that is, to an increased diffusion constant).

4. MCRG estimation of the exponent ν for the TSAW

In the year 1979 Swendsen suggested a very general and direct method to simulate the fixed point governing the critical behaviour of a given model and to estimate the corresponding critical exponents from the renormalisation group transformation at this fixed point. In order to obtain a simulation of the fixed point model one only has to simulate the original model at the critical point (this is $K \to 1$ for kinetic walk models) and to carry out a length rescaling which then leads to typical configurations of the rescaled model. For a sufficiently large rescaling factor one automatically obtains a simulation of the exact fixed point without any truncation of the number of parameters. The only purpose of the considerations in the preceding section is to reduce this rescaling factor to a value as small as possible in order to obtain useful results with a reasonable amount of computer time.

In practice, one considers the configurations obtained from a Monte Carlo simulation for the original model by rescaling by the factors $s_i = 2^i$ ($i = 1$-I) and estimates certain average values. The case of kinetic walk models is particularly easy because one knows the exact critical parameter $K_c = 1$ and one has a relation between the fugacity parameter K of the model and the average value of the number of steps. Therefore, one only has to measure the average value $\langle N_i \rangle$ at each rescaling level i which fixes the corresponding fugacity K_i via equation (3). At each rescaling level i one obtains $\nu_i(K_i)$ from (compare equation (5))

$$\nu_i(K_i) = \ln 2/\ln[(1-K_{i+1})/(1-K_i)] \tag{9}$$

and subsequently one has to carry out the extrapolation $K_0 \to K_c = 1$ ($K_0 \to 1$ implies $K_i \to 1$ for all i). Because with increasing rescaling level i the rescaled model approaches the fixed point the exponents ν_i converge to the exact value of ν for $i \to \infty$. In practice, this limit has to be reached for sufficiently small values of i and this is the only reason for starting with the approximate fixed point of table 1 instead of starting with the TSAW itself.

In order to estimate the exponent ν of the TSAW the model of table 1 has been simulated, and the values of the rescaled fugacities K_i have been estimated for the rescaling levels $i = 1$–5 and for various values of K_0. Altogether simulations for 50 different values of K_0 have been carried out, each simulation consuming some 90 min of CPU time on a Sperry 1100/82 computer.

Figure 5 shows ν_i as a function of $1/\langle N_i \rangle$ for the rescaling level $i = 3$. Up to surprisingly large values of $1/\langle N_i \rangle$ the result of a linear extrapolation ($1/\langle N_i \rangle \to 0$) changes only very slowly with the upper boundary for the values of $1/\langle N_i \rangle$ considered (compare figure 2 for the TPM). In a more accurate extrapolation procedure one has to be very careful for two reasons. In the first place, $1/\langle N_i \rangle$ should be sufficiently small in order to be in the asymptotic range. On the other hand, $1/\langle N_i \rangle$ should not be too small, because otherwise large occupation numbers occur. However, the approximate fixed point has been derived only for small values of n and it is not clear whether the approximation of table 1 is sufficiently accurate for larger occupation numbers. Indeed, one observes for small values of $1/\langle N_i \rangle$ a deviation from the linear relation between ν_i and $1/\langle N_i \rangle$. This is a rather crucial point: simulations for smaller values of $1/\langle N_i \rangle$

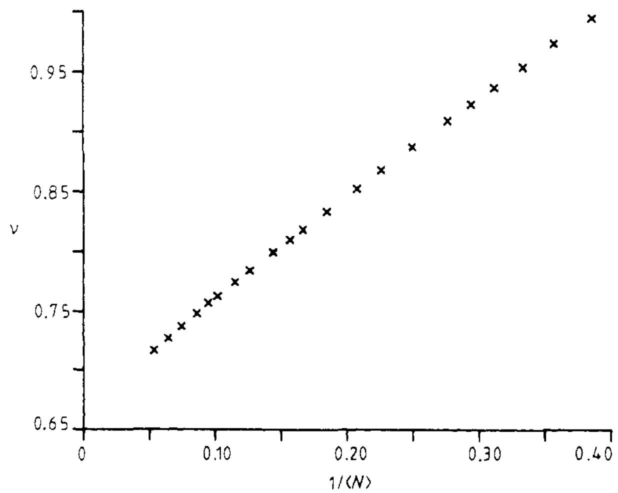

Figure 5. MCRG results for the exponent ν_i plotted against reciprocal number of steps $1/\langle N_i \rangle$ at the rescaling level $i = 3$ (the error bars are smaller than the size of the symbols).

do not always lead to a better extrapolation because our approximation to the fixed point may become worse for the correspondingly larger occupation numbers. Then, it might be necessary to go to much higher rescaling levels i in order to see the true fixed point behaviour (a bad approximation to the fixed point possibly approaches the exact fixed point as slowly as the TSAW itself).

Therefore, the evaluation of the results has been carried out in two steps. To begin with it was checked for different values of ε_i, to see whether a linear relation between ν_i and $1/\langle N_i\rangle$ holds for $\varepsilon_i < 1/\langle N_i\rangle < 0.2$ (more precisely, quadratic effects in $1/\langle N_i\rangle$ have also been taken into account by second order linear regression). The quantity ε_i has been chosen so large that within the statistical error bars no deviation from such a relation could be found. Typical values for ε_i are some 0.05, that is $\langle N_i\rangle \approx 20$, and consequently the values of the occupation number n correspond roughly to the range for which the approximate fixed point of table 1 has been derived.

In the second step a second order linear regression has been carried out where the data for $1/\langle N_i\rangle$ in the range from ε_i to m_i for several values of the upper bound m_i have been taken into account. The results are summarised in table 2. For the rescaling

Table 2. MCRG estimates for the exponent ν of the TSAW for different rescaling levels i. The values in the table have been obtained by linear extrapolation ($1/\langle N_i\rangle \to 0$), where only the results for $1/\langle N_i\rangle$ smaller than m_i have been taken into account.

i	$m_i = 0.30$	$m_i = 0.25$	$m_i = 0.20$
1	0.672 ± 0.001	0.670 ± 0.001	0.668 ± 0.001
2	0.665 ± 0.001	0.664 ± 0.001	0.663 ± 0.001
3	0.672 ± 0.001	0.671 ± 0.001	0.670 ± 0.002
4	0.671 ± 0.001	0.670 ± 0.001	0.669 ± 0.002

levels $i = 3$ and $i = 4$ the results are completely consistent, whereas the data for $i = 2$ are clearly different from the data for $i = 3$ and 4. Taking into consideration the tendency of the results to decrease with m_i and assuming that the asymptotic result is reached for $i = 3$ and 4 one obtains from table 2 the final estimate for the exponent $\nu = 0.665 \pm 0.004$ for the TSAW which is in good agreement with the direct Monte Carlo results $\nu = \frac{2}{3} \pm 0.003$ (Rammal *et al* 1984), $\nu = 0.67 \pm 0.01$ (Bernasconi and Pietronero 1984) and the possibly exact value $\nu = \frac{2}{3}$.

5. Summary

A canonical renormalisation rule for one-dimensional kinetic walk models has been described in the context of the TPM for which the exact renormalisation group transformation has been determined and discussed. The flow diagram of the TPM shows two fixed points corresponding to the self-avoiding walk with end-to-end distance exponent $\nu = 1$ and to the pure random walk with $\nu = \frac{1}{2}$.

For the TSAW an approximate expression for the fixed point governing its asymptotic behaviour has been obtained by Monte Carlo renormalisation according to the canonical rule. To this end the behaviour of the TSAW upon length rescaling has been studied in detail. The fixed point is characterised by typically very small self-avoidance parameters and a suppression of turning points (that leads to an increased diffusion constant).

Following the general ideas of Swendsen (1979), the approximate expression for the fixed point has been used as a starting point for a MCRG estimation of the exponent ν for the TSAW (the TSAW itself has not been chosen as a starting point since it is known to approach only very slowly the corresponding fixed point). The extrapolation of the results to long walks is very crucial because the approximate fixed point has been determined only for small occupation numbers. As a final result $\nu = 0.665 \pm 0.004$ has been obtained.

Acknowledgments

The author is indebted to Professor J Honerkamp for stimulating discussions. The numerical calculations have been done on the University of Freiburg's Sperry 1100/82 computer.

References

Amit D, Parisi G and Peliti L 1983 *Phys. Rev.* B **27** 1635
Bernasconi J and Pietronero L 1984 *Phys. Rev.* B **29** 5196
Bulgadaev S A and Obukhov S P 1983 *Phys. Lett.* **98A** 399
Byrnes C and Guttman A J 1984 *J. Phys. A: Math. Gen.* **17** 3335
de Gennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca: Cornell University Press)
de Queiroz S L A, Stella A L and Stinchcombe R B 1984 *J. Phys. A: Math. Gen.* **17** L45
Duxbury P M and de Queiroz S L A 1985 *J. Phys. A: Math. Gen.* **18** 661
Duxbury P M, de Queiroz S L A and Stinchcombe R B 1984 *J. Phys. A: Math. Gen.* **17** 2113
Family F and Daoud M 1984 *Phys. Rev.* B **29** 1506
Kogut J and Wilson K G 1974 *Phys. Rep.* **12** 76
Kremer K, Baumgärtner A and Binder K 1981 *Z. Phys.* B **40** 331
Ma S-k 1976 *Phys. Rev. Lett.* **37** 461
Nakanishi H and Family F 1984 *J. Phys. A: Math. Gen.* **17** 427
Obukhov S P 1984 *J. Phys. A: Math. Gen.* **17** L7
Obukhov S P and Peliti L 1983 *J. Phys. A: Math. Gen.* **16** L147
Öttinger H C 1985a *J. Phys. A: Math. Gen.* **18** L299
—— 1985b *J. Phys. A: Math. Gen.* **18** L363
Peliti L 1984a *Phys. Rep.* **103** 225
—— 1984b *J. Physique* **45** L925
Pietronero L 1983 *Phys. Rev.* B **27** 5887
Rammal R, Angles D'Auriac J C and Benott A 1984 *J. Phys. A: Math. Gen.* **17** L9
Redner S and Reynolds P J 1981 *J. Phys. A: Math. Gen.* **14** 2679
Stanley H E, Reynolds P J, Redner S and Family F 1982 *Real-Space Renormalisation* ed T W Burkhardt and J M J van Leeuwen (Berlin: Springer) pp 169-206
Stella A L, de Queiroz S L A, Duxbury P M and Stinchcombe R B 1984 *J. Phys. A: Math. Gen.* **17** 1903
Swendsen R H 1979 *Phys. Rev. Lett.* **42** 859
—— 1982 *Real-Space Renormalization* ed T W Burkhardt and J M J van Leeuwen (Berlin: Springer) pp 57-86